Comment on "Control of hyperchaos"

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In this Comment we show that the adaptive adjustment mechanism proposed by Shouliang, Shaoqing, and Hengqiang [Phys. Rev. E **64**, 056212 (2001)] does not ensure that the feedback system will converge to the desired orbit as the real system is, in general, not known.

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Ott, Grebogi, and Yorke (OGY) [1] have proposed an efficient method of chaos control using a stabilizing feedback control law formulated in terms of a parameter vector of the system accessible for control. The control law is made active when the system trajectory is near the neighborhood of the desired orbit or fixed point. A variation of the OGY method is given in [2] for the problem of stabilizing an unstable orbit embedded in a space of dimension $N \geqslant 1$ near a fixed point at which the dimension of the unstable manifold is also N. The system under consideration in [2] is represented by the map $T: x_n \to x_{n+1}$,

$$x_{n+1} = F(x_n, p), \tag{1}$$

where $F: \mathbb{R}^N \times \mathbb{R}^1 \to \mathbb{R}^N$, $x \in \mathbb{R}^N$ is the state space vector and $p_n \in \mathbb{R}^{N_u}$ is a parameter vector that can be externally modified. For flows, map (1) is a Poincaré map. Let x_*^0 be the fixed point of map (1) with $p=0$. Let J be the Jacobian matrix of the map (1) with $p=0$ evaluated at the fixed point x_*^0. It is assumed that proper coordinate changes have been made so that x_*^0 is the origin of the N-dimensional space and that the eigenvectors of the Jacobian matrix are along the coordinate axes of the space. As all the eigenvalues of J have modulus greater than 1, the implicit function theorem can be used to assert that the map (1) with small parameter p has a fixed point x_* in the neighborhood of x_*^0.

The control law in p proposed in [2] is of the form

$$p_n = P^{-1}(J-I)^{-1}(J-kI)x_n \tag{2}$$

with $P = (\partial x_* / \partial p)_{p=0}$, $-1 < k < 1$, I an $N \times N$ identity matrix, and p renamed as p_n to indicate that the parameter adjustment is in the nth iteration of the map. With the parameter adjusted according to Eq. (2), iterations of the map (1) converge to the fixed point x_*^0 monotonically, and therefore the stabilization is achieved.

In [3] the authors claimed that, differing from the well-known OGY method and its variants, an accessible parameter for control is not required to force the system solution to converge to their original periodic orbits. Their method comes from the original adaptive adjustment mechanism (AAM) [4]. The AAM method considers an N-dimensional nonlinear discrete system defined by

$$x_{n+1} = F(x_n), \tag{3}$$

where $F: \mathbb{R}^N \to \mathbb{R}^N$ and x is as already defined. A modified system is then constructed from Eq. (3),

$$x_{n+1} = \tilde{F}(x_n) := (1-\gamma)F(x_n) + \gamma x_n, \tag{4}$$

where γ is a positive controlling parameter introduced.

Note that systems F and $\tilde{F}$ share exactly the same set of fixed points and there exists for each and every point of F and $\tilde{F}$ the following one-to-one correspondence between their eigenvalues $\tilde{\lambda}_j = (1-\gamma)\lambda_j + \gamma$, $j = 1,2,\ldots,N$. Therefore, a fixed point of $\tilde{F}$ is stable if and only if $\max|\lambda_j| < 1$ and the eigenvalues $\tilde{F}$ can be adjusted by suitable choice of the adaptive parameter γ.

In the method described in [3], in order to stabilize the desired fixed point, the following control strategy is proposed:

$$x_{n+1} = F(x_n) + M(F(x_n) - x_n), \tag{5}$$

where M is an $N \times N$ matrix to be determined. Although Eq. (5) takes the form of the AAM, the matrix M is not restricted to be a diagonal matrix. As in the AAM mechanism, systems (3) and (5) share exactly the same set of fixed points. For easy reference, the main steps of the control procedure presented in [3] are given. Let an infinitesimal deviation of x_n from x_f be $\delta x_n = x_n - x_f$. Taking a linear approximation of Eq. (5) in a neighborhood W of the fixed point x_f yields

$$\delta x_{n+1} \approx J\,\delta x_n + M(J-I)\,\delta x_n, \tag{6}$$

where J is the Jacobian of the diagonal system (3) at x_f and I is the $N \times N$ identity matrix. In practice, the matrix J can be experimentally obtained via the well-known embedding technique. The control objective is to make $\lim_{n\to\infty}|\delta x_n| \to 0$. For this aim it is set

$$\delta x_n = \sigma(n-n_0)\,\delta x_{n_0}, \tag{7}$$

where $\delta x_{n_0} = x_{n_0} - x_f$, $x_{n_0} \in W$, and $\sigma(n)$ is a scalar function of n, which satisfies $\sigma(n) \to 0$, as $n \to 0$. Without loss of generality, one may choose $n_0 = 0$ hereafter. Substituting Eq. (6) into Eq. (7) and eliminating δx_{n_0} yields

$$M = \left(\frac{\sigma(n+1)}{\sigma(n)}\right)(J-I)^{-1}, \tag{8}$$

where it is assumed that the matrix $(J-I)$ above is invertible and $\sigma(n)$ is defined as

$$\sigma(n) = \gamma^n, \tag{9}$$

where γ is a constant and $\gamma\in(-1,1)$. Making use of Eq. (9), the matrix M now becomes

$$M=(\gamma I-J)(J-I)^{-1}. \tag{10}$$

Although it is clear that no accessible parameter is used in this method, it is not true that the solution will converge to the original periodic orbit. If we compare Eq. (5) and

$$x_{n+1}=F(x_n)+u_n, \tag{11}$$

we can recognize the following feedback control law:

$$u_n=MF(x_n)-Mx_n. \tag{12}$$

From Eq. (3), we see that the control signal u at the time n depends on the value of the state vector x at the time $n+1$, which is not available. Although we can make an approximation of it using the equation of the map, the solution of the feedback system will not converge to the desired orbit as the real system is, in general, not completely known. As the experimental system approaches the real system, the solution will, in general, approach the original periodic orbit.

As an example, consider a system represented by the same two-dimensional map, discussed in [3],

$$x_{n+1}=1-a(x_n^2+y_n^2)+p,$$
$$y_{n+1}=-4x_ny_n+q, \tag{13}$$

where $p=q=0$ and $a=2$.

The system possesses a fixed point at $(-0.25,0.75)$. For this fixed point we find

$$J=\begin{bmatrix}1 & -3\\ -3 & 1\end{bmatrix}.$$

Choosing $\gamma=0.5$ in Eq. (9), we calculate the matrix M required for the control,

$$M=\begin{bmatrix}-1 & \frac{1}{6}\\ \frac{1}{6} & -1\end{bmatrix}.$$

Suppose that F required in Eq. (12) was experimentally obtained and it does not correspond exactly to Eq. (13). Suppose $a=2.1$ in Eq. (13). Then, using this fact and the calculated M, Eq. (11) becomes

$$x_{n+1}=1-2(x_n^2+y_n^2)-[1-2.1(x_n^2+y_n^2)-x_n] + (-4x_ny_n-y_n)/6,$$

$$y_{n+1}=-4x_ny_n+[1-2.1(x_n^2+y_n^2)-x_n]/6-(-4x_ny_n-y_n). \tag{14}$$

Computational simulations show that it will converge to $(-0.1376,0.7230)$, which is different from the expected fixed point $(-0.25,0.75)$. As the parameter a in the experimental F approaches 2.0, the fixed point will approach $(-0.25,0.75)$.

In the method described in [2], using a parameter perturbation as in Eq. (2), the system will converge to the desired fixed point even if the matrix J required for the control is experimentally obtained and does not correspond exactly to the real one. To clarify how this happens, we take the same system above and calculate the control law for p and q for stabilizing the system to the case of the fixed point $(-0.25,0.75)$, as in [3]. First, we obtain the matrices J and P required for the control law (2),

$$J=\begin{bmatrix}1 & -3\\ -3 & 1\end{bmatrix},$$

$$P=\begin{bmatrix}0 & \frac{1}{3}\\ \frac{1}{3} & 0\end{bmatrix}.$$

Suppose that the matrix J was obtained experimentally and does not correspond to the one calculated above but it is given by

$$J=\begin{bmatrix}1 & -4\\ -4 & 1\end{bmatrix}.$$

Using $k=\frac{3}{4}$ in Eq. (2), the control is of the form

$$p_n=-3\left(\frac{1}{16}x_n-y_n+\frac{49}{64}\right),$$

$$q_n=3\left(x_n+\frac{-1}{16}y_n+\frac{19}{64}\right). \tag{15}$$

Computational simulations show that the system will converge to the desired fixed point $(-0.25,0.75)$.

Suppose now that the matrix P was also experimentally obtained and is given by

$$P=\begin{bmatrix}0 & \frac{2}{7}\\ \frac{2}{7} & 0\end{bmatrix}.$$

Using both the experimental matrices P and J, the control law is now given by

$$p_n=-\frac{7}{2}\left(\frac{1}{16}x_n-y_n+\frac{49}{64}\right),$$

$$q_n=\frac{7}{2}\left(x_n+\frac{-1}{16}y_n+\frac{19}{64}\right). \tag{16}$$

With this control law, the system will converge to $(-0.1797,0.8203)$, which is different from the original unstable fixed point $(-0.25,0.75)$.

The control law proposed in [3] relies on a linearization of the map, using its Jacobian matrix J, and on an estimative of the mapping function. The method [2] also relies on a linearization of the map but uses a parameter perturbation of the unstable fixed point to construct a stable path to guide the system to the target fixed point.

To conclude, in [3] incomplete knowledge of the mapping function leads to convergence to a fixed point that is different from the target, and in [2] perfect knowledge of the mapping function is not needed to achieve the target, but errors in the parameter matrix P are not allowed.

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